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A study of the energy spectra of muons underground

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Abstract. The energy spectra of atmospheric muons at sea level and underground have been calculated from assumed forms of the primary spectrum, the mass composition and the interaction model, and from various depth–intensity relations at great depths. The mean muon energies in the vertical direction at a depth of 7500 m.w.e. have been derived and are found to range from 286 GeV to 393 GeV according to the assumptions used.

The energy spectra of neutrino-induced muons underground have also been calculated. The mean energies are found to be much lower than for those of atmospheric origin, being typically in the region of some tens of GeV. It is shown that the rate of muons is sensitive to the mass of the (hypothetical) intermediate boson, and the mean muon energy is largely dependent on the saturation energy of an assumed linearly increasing cross section for the inelastic neutrino interaction. It is shown that more accurate measurements of both the rate and the mean energy of the neutrino-induced muons will provide useful information on the neutrino cross sections at very high energies.

1. Introduction

Neutrinos are produced in the atmosphere from the decay of the secondaries of the nuclear interaction of the cosmic-ray primary particles with air nuclei. Both muon and electron neutrinos are present. Several experiments are currently in progress to detect muons deep underground in order to study the characteristics of neutrino interactions at very high energies (Menon *et al.* 1967 b, Reines *et al.* 1966, Bergeson *et al.* 1966). These experiments have been designed to record muons initiated by muon neutrinos because, in contrast to electron neutrinos, the great penetration of muons yields a large target thickness in the surrounding rock for their production. Some of the high-energy muons of atmospheric origin can also penetrate to great depths, and a distinction between the two 'types' of muons is therefore of prime importance in any attempt to interpret the experimental results. General distinctions can be made from the fact that muons of atmospheric origin make only small angles to the vertical, whereas the muons at large angles to the vertical and those arriving from below the horizon are most likely to be induced by neutrinos.

The study of the energy spectra of these two types of muons is of interest because the atmospheric muons are closely related to the primary spectrum, the mass composition and the characteristics of the high-energy interaction process, whilst the neutrino-induced muons provide information on the behaviour of the high-energy neutrino cross sections and the nature of the secondaries from the interactions.

In the present work the relationship between the various components in the derivation of the underground muon spectra from a primary energy spectrum is first discussed. The energy spectra of atmospheric muons both at sea level and at a depth of 7500 m.w.e. standard rock (i.e. the location of the neutrino experiment at Kolar Gold Fields, India) are then calculated for the assumed primary nucleon spectra and for the adopted depth–intensity relations of muons at great depths. The mean muon energies for various assumptions are also given. After a brief review of the cross sections for various neutrino interaction processes, the energy spectra of neutrino-induced muons underground are calculated. It is shown that the rate of neutrino events and the mean muon energies are closely related to the assumptions about the neutrino cross sections.

A preliminary report of the work has been given by Craig *et al.* (1967) and somewhat similar calculations on neutrino-induced muons, but restricted to energies below 20 GeV, have been reported by Cowsik *et al.* (1966).

2. The various aspects concerning the muon spectra

2.1. *Relationship between the various spectra*

The relationship between the various components in the derivation of the energy spectra of muons is summarized in figure 1.

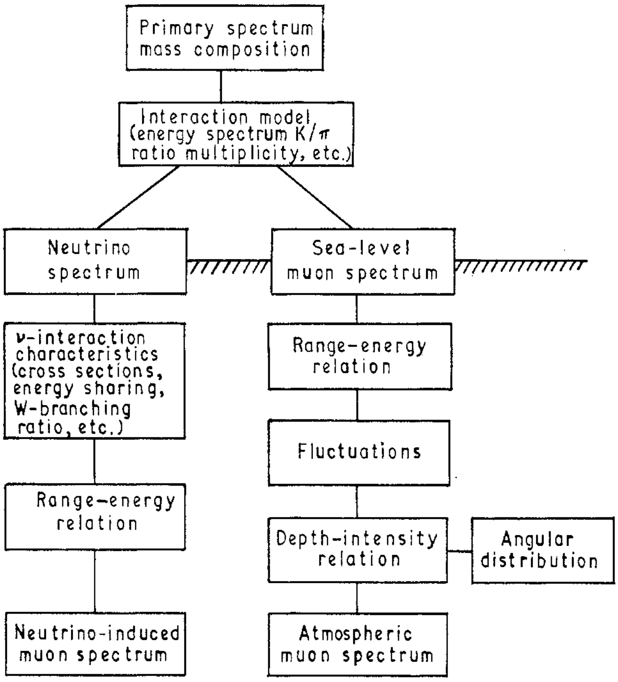

Figure 1. The relationship between the various components, from the primary spectrum to the underground muon spectra.

In order to derive an expected energy spectrum of the atmospheric muons underground, either the sea-level energy spectrum or the depth–intensity relation is required. The latter has been directly measured in underground experiments to a depth of about 8500 m.w.e. with decreasing accuracy (Menon *et al.* 1967 a). At great depths, for example 7500 m.w.e., where the Kolar Gold Fields (K.G.F.) experiment is situated, information is also available from measurements of the angular distribution of atmospheric muons.

In proceeding either from the sea-level spectrum or the depth–intensity relation, corrections are necessary for fluctuations in energy loss when the energies involved are high (see the previous paper, Osborne *et al.* 1968), and these corrections have been made.

The energy spectra of neutrino-induced muons can be calculated from a knowledge of the neutrino spectrum, the details of the high-energy neutrino cross sections and the range–energy relation of muons in rock which constitutes the effective target for neutrinos.

2.2. *Sea-level muon spectrum*

For energies below 10^3 GeV direct measurements have been made with magnetic spectrographs, and this spectrum, together with the range–energy relation for muons, can be used to derive the low-energy end of the underground spectrum at comparatively shallow depths.

At energies above 10^3 GeV no direct measurements have been made of the muon spectrum at sea level, and it must be inferred from indirect methods. In general, there are two possible methods to achieve this: (i) to work down from an assumed primary cosmic ray spectrum and an interaction model, or (ii) to use the measured depth–intensity relation, together with the range–energy relation and corrections for range fluctuations.

The latter method has been used by a number of workers (e.g. Osborne *et al.* 1964) and the sea-level spectrum has been estimated to some 7000 GeV, which corresponds to an average muon range of 6300 m.w.e. However, it is at the depths where the neutrino experiments are situated (7500–8500 m.w.e.) that a knowledge of the expected energy spectra of atmospheric muons is most useful, and this can only be derived from a sea-level spectrum extending to much higher energies.

In order to derive the expected muon spectrum at 7500 m.w.e., either the form of the measured depth–intensity curve must be extrapolated to some 15 000 m.w.e., or a sea-level muon spectrum extending to 10^5 GeV must be deduced from a primary spectrum. The results of both methods are given in this paper.

2.3. *The neutrino spectra*

Knowledge of the neutrino spectra is necessary in any attempt to derive the expected spectra and rates of neutrino-induced muons which will in turn provide information on the neutrino cross section at high energies. The fact that these cross sections are extremely small implies that the neutrino spectra are essentially the same at sea level and at great depths, and indeed are the same for upward and downward moving neutrinos.

The neutrino spectra have been calculated by a number of workers (e.g. Osborne *et al.* 1965). For neutrino energies up to 10^3 GeV the general procedure is to deduce the spectra of the parent pions and kaons (assuming a value for the ratio of kaons to pions in high-energy interactions) from the measured energy spectrum of muons at sea level and then to deduce the neutrino spectra. The muon spectrum is also calculated as a function of depth in the atmosphere and its contribution to the neutrino intensity from the μ–e decay is computed.

For energies above 10^3 GeV the measured muon spectrum at sea level can no longer be taken as a datum, and the necessary production spectra of pions and kaons are deduced from an adopted primary spectrum and interaction model. Thus the neutrino spectra extending to some 10^4 GeV have been obtained.

Finally, results are presented of the energy spectra of the muons to some 10^3 GeV produced in the earth in neutrino collisions for various assumptions about the neutrino interaction characteristics.

3. The spectra of the atmospheric muons

3.1. *Derivation from an assumed primary spectrum*

3.1.1. *The primary spectrum.* For energies above some 10^4 eV the energy spectrum of primary cosmic rays has been derived from the size spectrum of extensive air showers. The spectrum so calculated refers to the primary energy per incident nucleus, whereas in order to calculate the resultant pion, kaon (and eventual muon) spectra the spectrum in terms of energy per incident nucleon is required. In order to make the conversion the mass composition of the primaries is required but, as is well known, this composition is not certain at the primary energies in question. The more favoured view is that there is a cut-off in the primary rigidity spectrum at about 3×10^{15} eV (e.g. Khristiansen *et al.* 1966) which can be attributed to galactic modulation and, as a result, the relative importance of heavy nuclei increases considerably at energies per nucleus above 3×10^{15} eV. However, this view cannot be regarded as a certainty.

In the present treatment alternative primary spectra are taken corresponding to the inclusion or exclusion of heavy nuclei. For the first spectrum the form given by Adcock *et al.* (1967), for the spectrum in terms of energy per nucleus, is taken and converted to energy per nucleon on the assumption that the mass composition below 10^{15} eV is as found at much lower energies, and as a result of modulation the heavy nuclei are all important above a few times 10^{15} eV. This spectrum is denoted by M in figure 2.

For the second spectrum the form given by Greisen (1966) in terms of energy per nucleus is taken and assumed to apply for protons alone, i.e. heavy nuclei are assumed to be negligible at all energies. This spectrum is denoted by U2 in figure 2. The equivalent spectrum found by assuming that the spectrum given by Adcock *et al.* for energy per nucleus can be used directly for energy per nucleon, i.e. neglecting heavy nuclei completely is also shown in figure 2, where it is denoted by U1. Since M and U2 represent the extreme

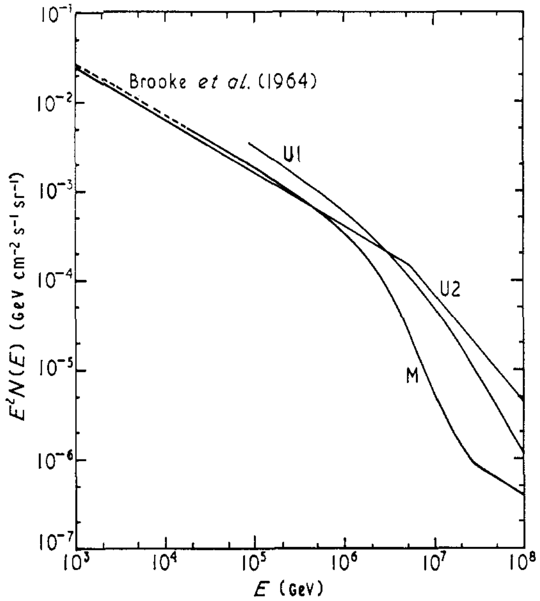

Figure 2. Comparison of primary spectra. The abscissa is in terms of energy per nucleon. In U1 and U2 it is assumed that the mass composition is essentially the same at all energies; in M galactic rigidity modulation is assumed and there is an increased fraction of heavy nuclei above 10^6 GeV.

shapes of the primary spectrum, calculations are made for them; the results for spectra derived from U1 will be intermediate.

3.1.2. *The derived muon spectra at sea level.* In the calculations of the sea-level muon spectra from the assumed primary spectra the method outlined by Brooke *et al.* (1964) has been followed. The empirical relation (to be referred to as the CKP relation) put forward by Cocconi *et al.* (1961) has been used for the energy spectrum of secondaries from nucleon–air-nucleus collisions. In this relation the number of pions emitted in the forward direction in the C system, $N(E_\pi)$, is given by

$$N(E_\pi)\, dE_\pi = \frac{A}{T_\mathrm{p}} \exp\left(-\frac{E_\pi}{T_\mathrm{p}}\right) dE_\pi$$

where E_π is the energy of the pion in the L system, A is the mean multiplicity of pions emitted in the forward direction in the C system and is assumed to vary as $E_\mathrm{p}^{1/4}$, where E_p is the primary energy and T_p is the mean pion energy. In our present treatment the contribution from kaons has also been included, the K/π ratio being assumed to be 20% at all energies (Osborne and Wolfendale 1964). The vertical sea-level muon spectra so derived have been normalized to the integral intensity given by Osborne *et al.* (1964) at 2000 GeV because of the possible uncertainties in the absolute intensity of the primary spectrum and in the multiplicity factor. These spectra are shown in figure 3 for muon energies up to about 10^6 GeV.

It should be pointed out that, although the CKP relation originally referred to nucleon interactions at machine energies (i.e. < 30 GeV), there is some evidence suggesting applicability to at least 10^5 GeV. This arises from the agreement between directly measured primary intensities and those derived from the measured muon spectrum at sea level and a model involving the CKP relation.

3.1.3. *The muon spectra underground.* From the derived muon spectra at sea level given in figure 3 and the survival probabilities derived by Osborne *et al.* (1968) the variation of muon intensity with depth has been calculated and is shown in figure 4 for depths greater

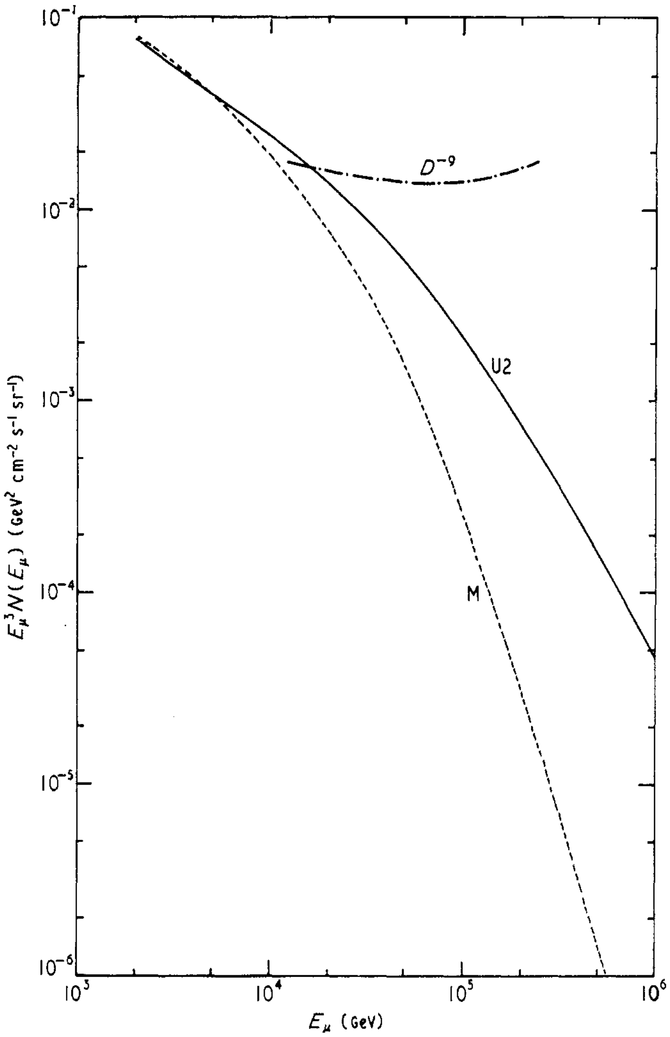

Figure 3. Differential muon spectra at sea level. U2 and M are derived from the corresponding primary spectra and D^{-9} from the depth–intensity relation $I_v(D) \propto D^{-9}$ for depths greater than 7500 m.w.e.

than 7500 m.w.e. At shallower depths the two derived depth–intensity curves are very close together, and differences in the muon intensities are significant only at very great depths. From the depth–intensity curves shown in figure 4 and the fluctuation corrections the energy spectra of atmospheric muons in the vertical direction at a depth of 7500 m.w.e. have been derived and are shown in figure 5.

3.2. *Derivation from an adopted depth–intensity relation*

3.2.1. *The depth–intensity relation.* This relation has been directly measured fairly accurately from underground experiments to depths of about 7000 m.w.e., below which the accuracy is poor. The sea-level muon spectrum derived from such a depth–intensity relation will extend to energies in the region of 10^4 GeV and will lie within the range covered by those derived from the M and U2 primary spectra. In order to extend the sea-level spectrum to higher energies so as to compare with those derived from the primaries, a depth–intensity relation at greater depth is required.

Menon *et al.* (1967 a) have shown that all the measured vertical intensities (which refer to depths shallower than 8500 m.w.e.) can be represented by an exponential variation which, extrapolated down to 15 000 m.w.e., is shown in figure 4. The validity of such an extrapolation to depths where no direct measurements have been made is of course uncertain.

Information on the depth–intensity relation is also available in the form of measurements of the angular distribution of atmospheric muons at great depths underground, notably in

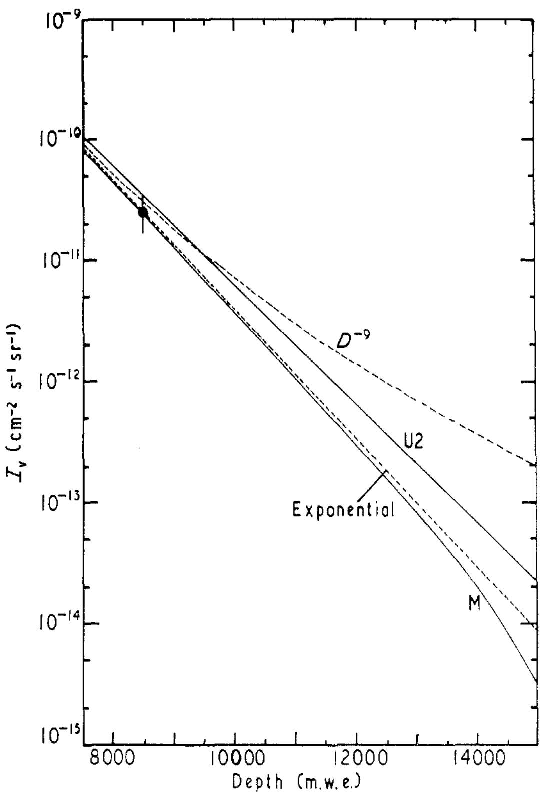

Figure 4. Depth–intensity variations at great depths.

the Kolar Gold Fields experiment at 7500 m.w.e., although at intermediate zenith angles, 30°–60°, difficulties arise in distinguishing between muons of atmospheric and neutrino origin.

An examination of the observed angular distribution in the Kolar Gold Fields experiment (Menon *et al.* 1967 b) shows poor agreement with the assumption of a nearly isotropic distribution of neutrino-induced muons, together with atmospheric muons following the expected exponential angular distribution. The prominent feature of the data is an apparent excess of particles in the range of projected angles 40°–50°, although the statistical precision of the data is still rather poor. If we discount explanations of the excess in terms of pions from local nuclear interactions or muons from Glashow resonance interactions, a more likely explanation is that the angular distribution of the atmospheric muons follows a flatter distribution.

Up to a zenith angle of 60° a $\cos^8 \theta$ variation is a tolerable fit to the data, and the corresponding depth–intensity relation is $I_v(D) \propto D^{-9}$, where D is the depth. This relation, normalized to the measured intensity at 7500 m.w.e., is shown in figure 4 where it is assumed to be valid to 15 000 m.w.e. The experimental limits on the muon intensity at 8500 m.w.e. given by the Case–Wits group (Reines *et al.* 1966) and shown in figure 4 are not sufficiently narrow to enable us to distinguish between the different depth–intensity curves.

3.2.2. *The derived muon spectra.* From the two adopted depth–intensity curves shown in figure 4 the corresponding muon energy spectra at 7500 m.w.e. have been derived and are shown in figure 5. The sea-level muon spectrum corresponding to the depth–intensity variation $I_v(D) \propto D^{-9}$ has also been derived for energies up to about 2×10^5 GeV, which corresponds to an average muon range of 15 000 m.w.e., and this is shown in figure 3.

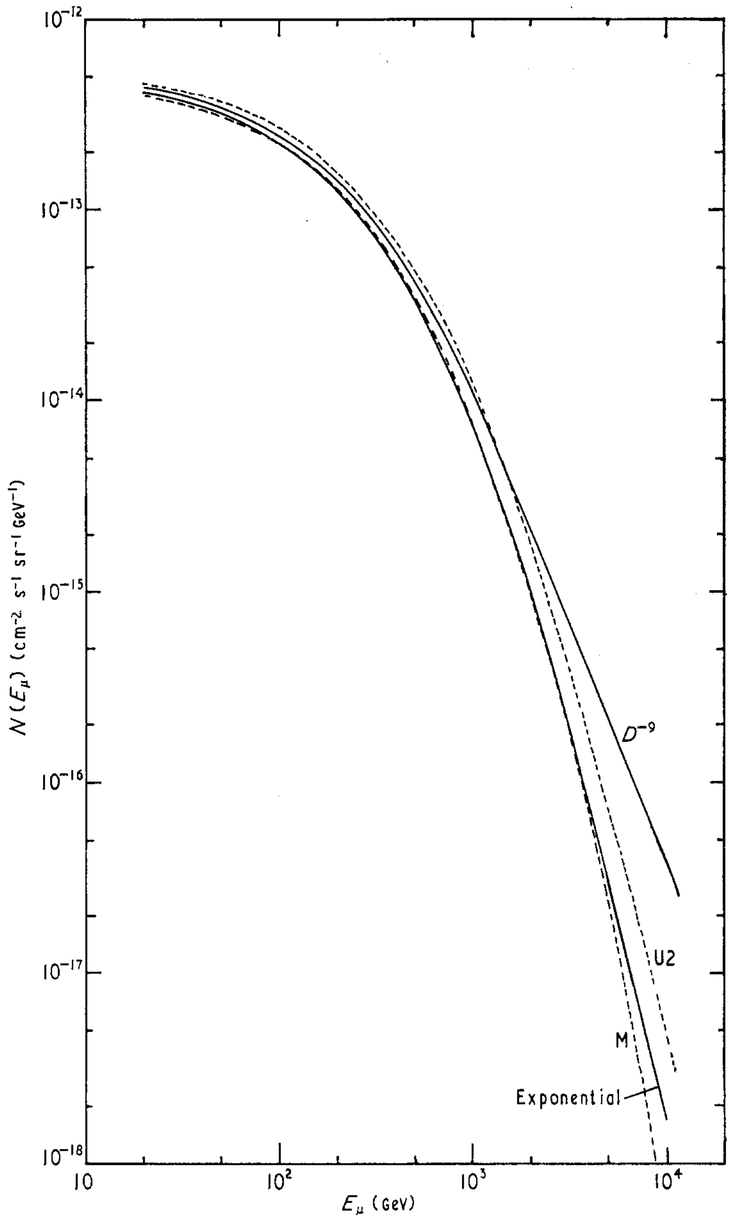

Figure 5. Vertical spectrum of atmospheric muons at 7500 m.w.e.

3.3. *Discussion and conclusion from atmospheric muon spectra*

The mean muon energies corresponding to the various muon spectra at 7500 m.w.e. shown in figure 5 are listed in table 1. It is clear that these energies are rather high, so that direct measurement is difficult.

From the depth–intensity curves shown in figure 4 mean muon energies at great depths have been calculated and their variation with depth is shown in figure 6, together with the mean energy calculated from the well-known depth–intensity relation at shallower depths. It is seen that an exponential depth–intensity relation would imply a constant mean muon energy at great depths.

It is interesting to note from figure 3 that the sea-level muon intensity at high energy derived from the D^{-9} depth–intensity relation is much higher than those derived from the

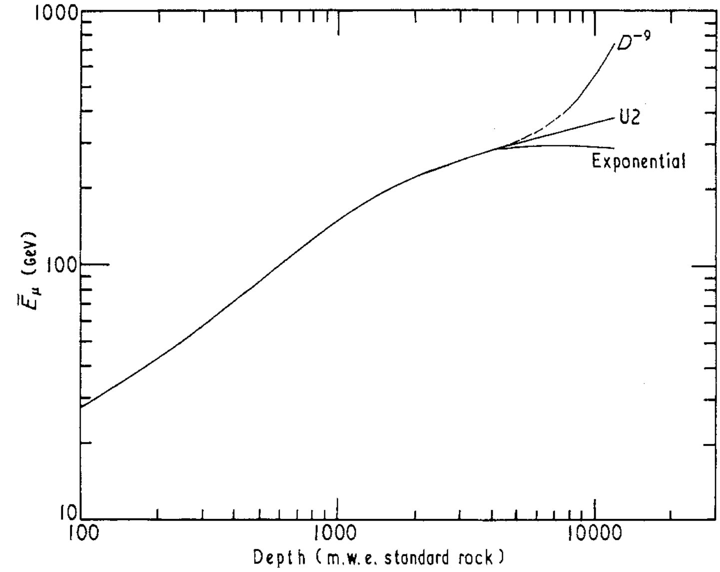

Figure 6. Variation of mean muon energy with depth. Below 5000 m.w.e. the curve has been derived from the measured depth–intensity relation. At greater depths the corresponding depth–intensity variations shown in figure 4 have been used. The broken curve is an interpolation.

Table 1. Mean muon energies expected at 7500 m.w.e. underground

(*a*) *Atmospheric muons in the vertical direction*

Depth–intensity relation	Primary spectrum	$\bar{E}_\mu$ (GeV)
exponential	—	287
D^{-9}	—	393
—	M	286
—	U2	336

(*b*) *Neutrino-induced muons in the horizontal direction*

Processes and cross sections	$\bar{E}_\mu$ (GeV)
(i) elastic $+\sigma_1 \propto E_\nu$ to 10 GeV, then constant	8·8
(ii) elastic $+\sigma_1 \propto E_\nu$ to 10^4 GeV, then constant	80
(iii) (i) + boson production, M_W = 1·8 GeV	28
(iv) (i) + boson production, M_W = 3·0 GeV	27
(v) (ii) + boson production, M_W = 1·8 GeV	60
(vi) (ii) + boson production, M_W = 3·0 GeV	76

various assumed primary spectra. If further observations deep underground should confirm that the angular distribution is indeed flatter than had been previously assumed and the mean energy is found to be correspondingly high, it would mean that the adopted model for the production and propagation through the atmosphere of the muon component, and that of its parents, is seriously in error. A model involving a change in high-energy characteristics has in fact been suggested by Adcock *et al.* (1967) in order to attempt to explain some features of extensive air showers. In one variant of the model a shortening of the mean free path is considered at primary energies above a few times 10^{15} ev, and from it an enhancement of the intensity of the high-energy muons results. The full implications of this model have yet to be worked out.

4. The spectra of neutrino-induced muons

4.1. *The neutrino cross sections*

The cross sections for the various types of neutrino interactions used in the calculations of the expected muon energy spectra are similar to those given by Menon *et al.* (1967 b). The following processes have been considered:

(i) elastic collision:
$\nu_\mu + \mathrm{N} \rightarrow \mu + \mathrm{N}'$

(ii) inelastic collision:
$\nu_\mu + \mathrm{N} \rightarrow \mu + \mathrm{N}' + \pi$'s etc.

(iii) production of intermediate boson W:
$\nu_\mu + \mathrm{Z} \rightarrow \mathrm{Z}' + \mu + \mathrm{W}$.

The cross sections for reactions (i) and (ii) have been measured up to about 10 GeV from machine experiments. Following these experiments, the elastic cross section is taken to be $0{\cdot}6 \times 10^{-38}\,\mathrm{cm}^2$ per n–p pair and independent of neutrino energy for $E_\nu > 1$ GeV. The inelastic cross section is $\sigma_i = 0{\cdot}3 \times 10^{-38}\, E_\nu\, \mathrm{cm}^2$ per nucleon for $E_\nu \lesssim 10$ GeV. Since no experimental data are yet available on the behaviour of σ_i at higher energies, it has been assumed to continue to increase linearly with energy until a saturation value. Calculations have been made, for different saturation energies, of the inelastic cross section and the results are presented here. The energy shared by the muon in reactions (i) and (ii) is assumed to be 90% and 67% of the neutrino energy, respectively, these being the values found at machine energies.

The existence of the intermediate boson W has not been established, but it has been shown that if it exists its mass is larger than 1·8 GeV. In the present calculations values of 1·8, 2·5 and 3·0 GeV have been taken, and the energy taken by the muon has been assumed to correspond to the 'near threshold' case given by Menon *et al.*, and a branching ratio of 40% has been assumed for the muon decay mode of the W.

4.2. *The expected energy spectra of neutrino-induced muons*

For energies below 1000 GeV the neutrino spectrum given by Osborne *et al.* (1965) has been used, and for higher energies it has been derived from the U2 primary spectrum. The differential energy spectra of neutrino-induced muons in the horizontal direction at 7500 m.w.e., derived from this neutrino spectrum and the neutrino interaction characteristics outlined above, are shown in figure 7. The effects of various values of M_W and of different saturation energies for the inelastic cross section are shown.

The mean energies of muons in the horizontal direction are listed in table 1 for a variety of assumptions regarding the neutrino cross sections. In these calculations a lower limit of 0·5 GeV has been adopted for the muon energy.

Since the neutrino intensity varies with inclination, calculations have also been made at other zenith angles and the energy spectra of muons from inelastic neutrino interactions at various zenith angles are given in figure 8. It can be seen that the difference in intensity increases with muon energy, as would be expected from the neutrino spectrum.

4.3. *Discussion*

From the energy spectra of neutrino-induced muons shown in figure 7 the relation between the neutrino event rate and the mean muon energy has been derived and is shown in figure 9. It is clear from this figure that both the rate and the mean energy are sensitive to the assumptions about the form of the neutrino cross sections. Furthermore, the event rate is largely dependent on the mass of the intermediate boson M_W, whereas the mean energy is sensitive to the saturation energy for an assumed linearly increasing inelastic cross section.

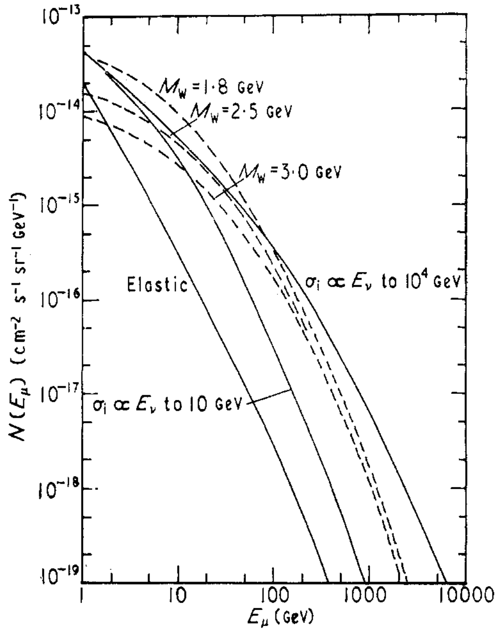

Figure 7. The horizontal spectra of neutrino-induced muons at 7500 m.w.e. for various types of neutrino interactions.

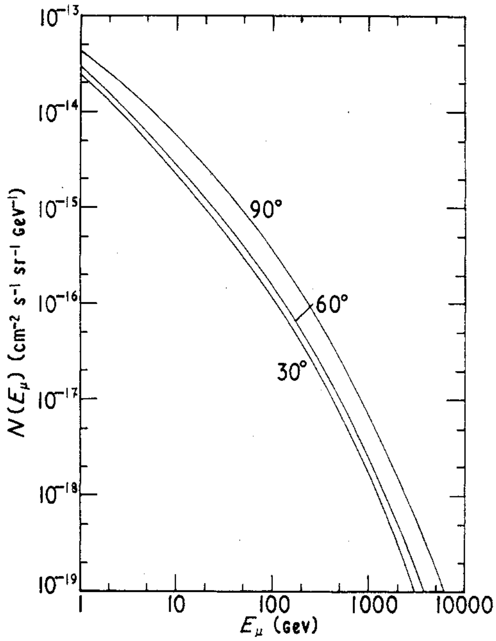

Figure 8. Comparison of the energy spectra of muons from inelastic neutrino interactions at zenith angles 90° (horizontal), 60° and 30°.

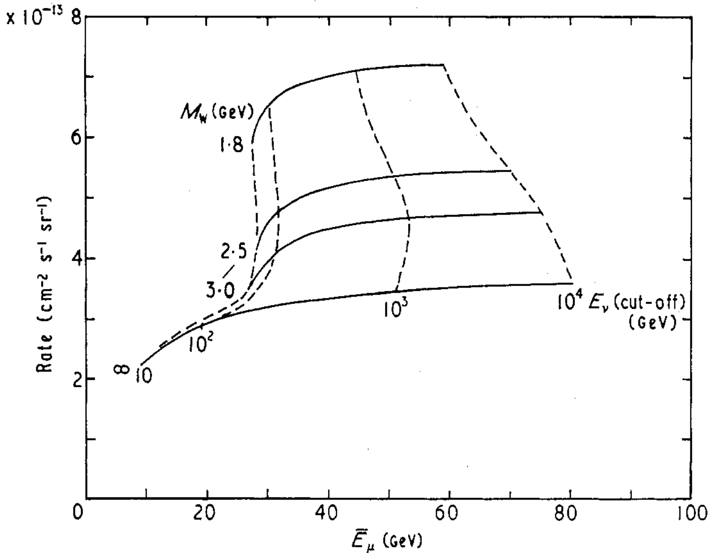

Figure 9. Relation between the rate and mean energy of neutrino-induced muons. The full curves represent the variations for a fixed mass M_W of the intermediate boson and a linear increase of the inelastic neutrino cross section to saturation at E_ν (cut-off). The broken curves are the variations for a fixed saturation energy and various values of M_W. (See the end of § 4.3 for a discussion of the consequence of a change in the mean fraction of energy taken by the muon.)

As yet the accuracy of the measurements of rate and mean muon energy in the neutrino experiments are too low for any firm conclusions to be made, but there is a suggestion from the Kolar Gold Fields experiment that the mean muon energy is probably less than a few tens of GeV. This would appear to indicate that the inelastic cross section saturates at not too high an energy. However, it has been pointed out by Lee (private communication) that the assumption made in § 4.1 that the muon takes 67% of the neutrino energy may not be valid at higher energies. If this fraction falls with increasing energy, the sensitivity of mean muon energy to saturation energy will be reduced considerably and the experimental results could well be consistent with a very high saturation energy (or no saturation at all).

5. Conclusions

The dependence of the energy spectrum of atmospheric muons on the form of the primary spectrum and mass composition has been calculated for an assumed interaction model. The mean muon energy deep underground is found to exhibit some sensitivity to these parameters, thus holding out the possibility of making an indirect study of them.

The energy spectrum of the underground neutrino-induced muons has also been calculated and it is found that the mean muon energy is lower than that for the atmospheric muons by a factor of the order of 10. This fact assists in the identification of these muons and suggests the possibility of deriving information about the characteristics of the high-energy neutrino interactions.

Acknowledgments

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